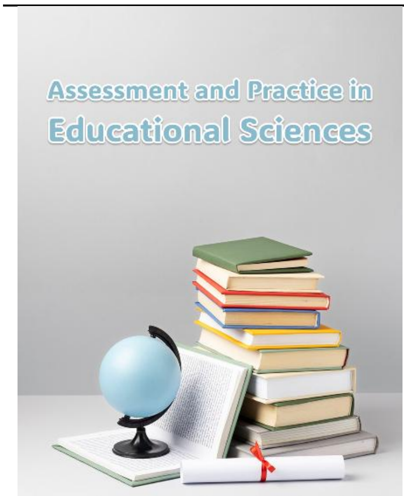
Assessment and Practice in Educational Sciences



Students' Aesthetic Experiences in Performing Realistic Activities of Geometric Transformations

1. Maryam. Shahmohammadi: Postdoctoral researcher, Department of Mathematics, Faculty of Science, Shahid Rajaee Teacher Training University, Tehran, Iran.
2. Elahe Aminifar*: Associate Professor, Department of Mathematics, Faculty of Science, Shahid Rajaee Teacher Training University, Tehran, Iran. (Email: shahmohamad.m@sru.ac.ir)

Article type:
Original Research

Article history:
Received 19 June 2024
Revised 23 October 2024
Accepted 08 November 2024
Published online 01 December 2024

How to cite this article:
Shahmohammadi, M., & Aminifar, E. (2024). Students' Aesthetic Experiences in Performing Realistic Activities of Geometric Transformations. *Assessment and Practice in Educational Sciences*, *2*(4), 1-18. https://doi.org/10.61838/japes.2.4.9

ABSTRACT
Diverse learners require education with a planned aesthetic to experience beautiful phenomena in mathematics. To do so, we designed an educational program based on aesthetic experiences for high school seniors in the eleventh grade for four months. We used Charmaz's constructivist grounded theory and a qualitative approach. We collected data through observation, recorded dialogues, and semi-structured interviews. We explain students' beliefs about the beauty of geometric transformations and the stages of logical thought required to perform associated activities. The findings indicate that the course based on the aesthetic experience of geometric transformations assists students in thinking about its nature aesthetically. The results also reveal aesthetic components as a medium for students to discuss mathematical ideas and develop logical thinking skills. We present the research outcome as a visual model to illustrate the relationship between geometric transformations, aesthetic experiences, logical thinking, and aesthetic beliefs.

Keywords: Aesthetic experience, Geometric transformations, Realistic activities, Belief, Logical thinking

Introduction

In contemporary educational discourse, there is increasing emphasis on cultivating 21st-century competencies—communication, collaboration, critical thinking, and creativity—to prepare students for global challenges and complex problem-solving tasks (1). One promising avenue to achieve these aims lies in integrating aesthetic experiences into educational design, particularly in the domain of mathematics. Despite a robust theoretical foundation, the practical implementation of aesthetic education within mathematics classrooms remains underdeveloped (2-4). Aesthetic experiences, when properly designed and embedded in instructional settings, have the potential to transform students' engagement with mathematical ideas, promoting both emotional resonance and logical sophistication (5-7).

The aesthetic dimension of learning, once considered a peripheral concern in STEM education, is now seen as integral to STEAM (Science, Technology, Engineering, Arts, and Mathematics) frameworks that prioritize holistic human development (8). Within this paradigm, mathematics is not only a logical system of abstract relations but also a deeply human endeavor that can evoke beauty, wonder, and appreciation (9, 10). The aesthetic potentials of mathematics—manifested in symmetry, harmony, simplicity, and structure—are particularly pronounced in geometry and its transformational processes, which combine spatial reasoning with perceptual elegance (11-13).

However, empirical studies suggest that aesthetic dimensions of mathematics are frequently overlooked or undervalued in everyday instruction (14, 15). This is unfortunate, as aesthetic appreciation can enhance motivation, curiosity, and even retention of mathematical knowledge (16, 17). As learners engage in mathematical activities that elicit affective and sensory responses, their cognitive engagement deepens, fostering more robust conceptual understanding (18, 19). Moreover, aesthetic learning may act as a democratizing force in mathematics education, allowing students with diverse talents and identities to connect with the subject in personally meaningful ways (20, 21).

This study is grounded in the philosophical and psychological roots of aesthetic theory, drawing from perspectives that regard beauty as both an experiential and cognitive phenomenon (22-24). Plato's notion of harmony and unity as the essence of beauty finds an echo in contemporary models of mathematical aesthetics, where patterns, structures, and logical coherence are seen as sources of aesthetic pleasure (25, 26). According to the fluency theory of aesthetic pleasure, beauty is experienced when cognitive processing becomes effortless due to familiarity, symmetry, or predictability (6). Thus, integrating aesthetic principles into mathematical instruction may lead to greater psychological fluency and emotional satisfaction, reinforcing learning outcomes.

The significance of geometry in fostering these experiences cannot be overstated. Geometric transformations—such as rotation, reflection, and translation—are not only foundational concepts in mathematics but also powerful vehicles for aesthetic expression. These transformations are deeply embedded in architecture, art, nature, and design, making them excellent entry points for blending the abstract with the concrete (27-29). When students engage with geometric transformations through realistic tasks—such as interpreting architectural motifs, tessellations, or symmetrical tilework—they encounter opportunities to see mathematics as a living, breathing discipline with relevance to everyday life (30, 31).

The educational value of realistic mathematics activities is well documented. The Realistic Mathematics Education (RME) approach, rooted in the philosophy of Freudenthal, views mathematics as a human activity closely tied to real-world contexts (32, 33). By situating mathematical concepts within meaningful, relatable scenarios, RME enhances student understanding and fosters deeper logical reasoning (28, 34). Moreover, when aesthetic experiences are embedded within such contexts, they can serve as catalysts for reflection, discussion, and the construction of mathematical meaning (35, 36).

This study is situated at the intersection of these theoretical insights. It seeks to explore how aesthetic experiences within realistic geometric transformation activities can influence high school students' beliefs about mathematics and their logical thinking skills. Parrish's (2) five principles of aesthetic instructional design—emphasizing rhythm, coherence, immersion, affect, and learner-centeredness—inform the educational interventions developed for this research. These principles align with Uhrmacher's (5) elements of aesthetic learning, including communication, sensory perception, and imaginative risk-taking. The design of the activities also draws from the aesthetic learning model proposed by Gadanidis et al. (7), which emphasizes surprise, paradox, and deep conceptual engagement.

Students' beliefs about the beauty of mathematics—both dispositional (long-held) and occurrent (emerging in the moment)—play a key role in shaping their engagement and problem-solving behaviors (37, 38). According to the Modified Strengthened Acquaintance Principle, aesthetic beliefs gain legitimacy only when grounded in direct or vividly imagined experience. In other words, experiencing mathematics through beautiful, coherent, and meaningful activities may transform students' implicit assumptions into explicit convictions about the aesthetic nature of the subject (3, 39). By tracking these belief transformations over time, this study aims to reveal how aesthetic exposure influences mathematical identity and epistemological stances (4, 40).

Further, aesthetic experiences are not merely affective—they are deeply cognitive. Logical thinking, as outlined by Dewey's classic triadic model (assignment, diagnosis, problem-solving), is interwoven with aesthetic dimensions in learning (24, 41not_in_list). Activities that are aesthetically engaging often elicit more rigorous reasoning, as students strive to uncover patterns, justify observations, and explore relationships among mathematical elements (13, 26). For example, when students engage in a task that involves reflecting tile fragments to form a symmetrical figure, the visual harmony they perceive becomes both a guide and a goal of their logical process. These aesthetic triggers can serve as metacognitive cues, prompting students to shift from trial-and-error strategies to principled reasoning (11, 42).

In terms of social learning, aesthetic experiences also facilitate collaborative discourse and critical dialogue—both vital for mathematical growth (6, 16). When students work in groups to discuss visually engaging problems, they are more likely to articulate their reasoning, challenge assumptions, and co-construct meaning. These social interactions help learners move from intuition to abstraction, often enabled by the aesthetic allure of the task itself (3, 43).

This study is particularly relevant in light of growing concerns about students' disengagement with mathematics due to perceptions of it being abstract, rigid, and disconnected from real life (15, 20). By embedding geometric transformation problems in culturally meaningful and artistically rich contexts—such as tiling, parquetry, or wall knots—this research aims to reclaim mathematics as a source of intellectual joy and aesthetic satisfaction. It also aligns with broader educational movements calling for more culturally responsive and ethically grounded curricula that reflect the diverse experiences and identities of learners (10, 44, 45). The current study aimed to explore the implementation of an educational model centered around aesthetic experiences within the classroom setting.

Methods and Materials

Study Design and Participants

The study model was designed based on the five principles proposed by Parrish (2009). The research took place over a period of four months, during which these principles were put into practice and subsequently analyzed.

This research is fundamental with a futuristic and decision-oriented perspective. The social constructivist paradigm considered as proposed by Creswell (2019), and the research design adopted is qualitative and interpretative. The research approach aligns with Charmaz's constructivist grounded theory (2006) and Braun and Clarke's thematic analysis method (2006; 2012) for collecting and analyzing data.

According to Creswell (2019), we selected the sample theoretically, homogeneously, and based on each participant's contribution to creating a substantive theory. The students were 49 females in the eleventh grade of a high school in Tehran. The first author taught them for an academic semester (four months) based on aesthetic experience. The age range of the students was between 16 and 18 years. Two students were 16 years old; forty students were 17 years old, and seven students were 18 years old. Twenty-five out of 49 students were allocated in five focus groups. The first author interviewed them according to the saturation of the information obtained during the implementation of the research, and we analyzed the data. Based on the research questions, we chose the axial symmetry (reflection) and rotation in geometric transformations from the eleventh-grade geometry textbook in Iran.

Data collection tools and procedure

We collected the data in two phases. During the first stage, we designed the lesson play for an educational course based on mathematical aesthetic experience within the context of geometric transformations. While conducting the teaching process, we collected data through various methods, including interactions, communication, and observation. In the second stage, we observed focus groups and used semi-structured interviews individually.

In the first stage, we designed twelve activities and selected four activities of geometric transformations considering the mathematical and aesthetic components according to the literature review and textbook content (Table 1). Then, we provided them with two experts related to the subject and inquired whether each activity could measure the related components (items). We used a 4-point scale related to each item and the proportion of items based on Waltz and Bausell (1981) to calculate the Content Validity Index for scales (S-CVI). Based on the feedback of both expert raters, who ranked 3 or 4 (indicating high or very relevant), we found an acceptable S-CVI value of 0.916. This value surpasses the threshold of 0.80, as Pollitt and Beck (2004) recommended.

Table 1. Content of activities based on aesthetic and mathematical components

Content	Activity	Behavioral objective	Cognitive domain (Bloom)	Aesthetic components	Mathematical components
Axial symmetry (reflection)	The clay tile	The student should obtain the form of a clay tile by using the components of the shape and the concept of symmetry.	Combining	Risk-taking and imagination (5)	Low floor, high roof (7)
	The game table	The student should know the concept of reflection and use its feature in problem-solving.	Analysis	Creative processes	Connection and communication
Rotation and application	The floor parquetry	The student should identify the rotation as a map and visualize the rotated form of a shape.	Understanding	Order and complexity (46)	Paradox (47)
	The knot wall	The student should apply the rotation features in doing real-life activities.	Application	Surprise and insight (7)	Deep nature, profound understanding and thinking (26)

Then we designed the educational activities based on the aesthetic experiences of geometric transformations (Figure 1) to create mathematical situations and interactive opportunities to discover the distinct aesthetic roles of geometric transformations.

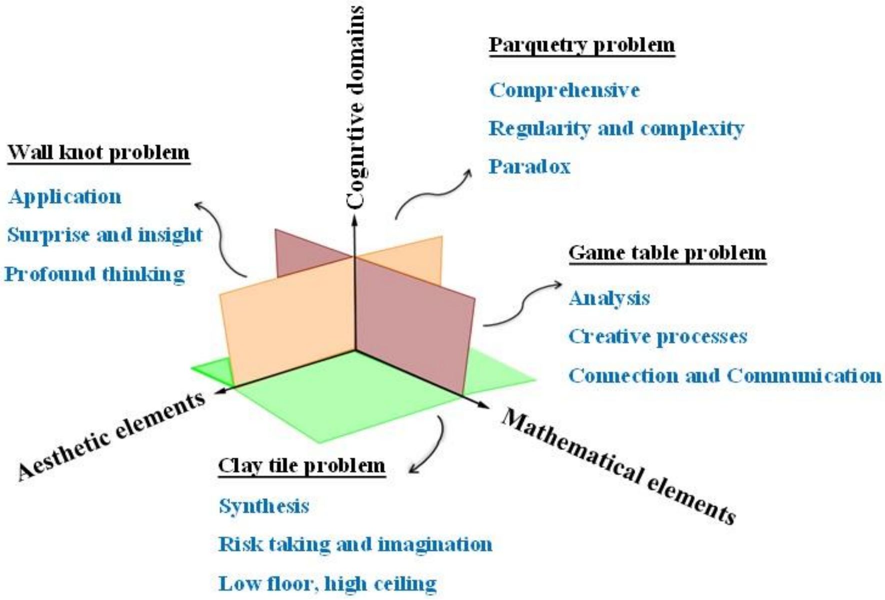


Figure 1. An educational design sample based on the aesthetic of geometric transformations

At this stage, a visualization of the lesson includes imagined interactions between the teacher and students that are closer to rehearsed-in-the-moment decisions than to predetermined plans and outcomes (35). We adopted this instructional design to cultivate an aesthetic appreciation of mathematics in students' learning experiences and explore their logical thinking skills while engaged in the learning process. When students feel secure, they may be able to experience surprise related to perceiving a mathematical problem as stunning (13); therefore, we assured them that the correctness or incorrectness of the solutions would not be marked during performing activities.

The four geometric activities constituted the core of 15 sessions, each lasting 90 minutes, including notetaking focusing on interaction, dialogue, cooperative observation, and documentation through photographs and videos.

Students initiated group dialogue throughout the implementation, sharing their ideas about the desired activity in their group with interaction and participation. Then, they explained the logical process of reaching their answers like a narrative to the teacher and other groups. We encouraged students to communicate their ideas as a narrative, to reveal the aesthetic components of geometric transformation and its distinct roles in the classroom in the form of storytelling, as Richman et al. (2019) used (6).

In the second stage of the study, the interview questions were designed by adapting the items from Brinkman's (2009) questionnaire and applying the authors' opinions (48). Some interview questions focused on the aesthetic nature of geometric transformations. For instance, "Are geometric transformations beautiful? How do you find beauty in geometry? In your opinion, what features of these activities related to geometric transformations make them beautiful?" Some questions were considered regarding affective aspects (emotions, beliefs, and enjoyment of learning) and cognitive factors (concepts and reasoning), such as "How did you feel when encountering each realistic geometric transformation activity? Which of these activities has been pleasant for you, and what features do you consider the reason for this pleasantness? Has the experience of encountering the activities of geometric transformations and solving them changed your belief about the beauty of mathematics? Have the aesthetic features of geometric transformations helped you understand the concepts and reach the solution to an activity? How do you evaluate the aesthetic experiences in performing the activities of geometric transformations in this educational course?"

The educational course also discussed the opportunities for students to participate and express their opinions while performing on the activities. The students found the opportunity to participate in the discussion and present solutions. In other words, the participants were provided with a space to express their beliefs, feelings, thoughts, and ideas.

Data analysis

Data was analyzed using the constructivist grounded theory, which includes *open*, *axial*, and *selective* coding phases. The educational course videos and students' notes while doing the activities were reviewed. The first author transcribed students' voices throughout the interviews, and we paid attention to the students' gestures and tones. We coded data using an open coding procedure according to the main categories of information. Then, we used thematic analysis, according to Braun and Clarke (2006), to obtain themes and categories during the axial coding phase (49, 50). We identified themes about common features and solutions that students attributed to realistic activities of geometric transformations. In the third phase, selective codes were formulated using sufficient evidence and data saturation.

Based on Silverman's method (2013), four coders are needed to check the reliability and intercoder agreement. Therefore, all four authors coded detailed interview field notes considering all pauses, minor verbal interventions, and students' handwriting. First, we read and coded several manuscripts independently (51). After that, we reviewed the codes, their names, and the text sections we had coded. Following achieving a consensus of 80% on the themes and categories (52), reviewed the coded texts and gave a "yes" or "no" answer to the question: "Have we all assigned the same code to this text?" Finally, we calculated the percentage of agreement among the four of us. Accordingly, we achieved an acceptable percentage of agreement on codes and themes at each stage of the process.

To ensure the credibility of the findings and interpretations, we used the "member checking" strategy. This strategy has been proposed as one of the most significant strategies to achieve credibility in qualitative research (53).

To implement this strategy into practice, we provide the focus groups with access to primary analyses, such as themes, categories, and descriptions. We asked them to comment and think about how accurate they are. In other words, the aim was to get students' feedback on the written analysis and identify any overlooked situations.

Findings and Results

To illustrate the role of aesthetic experiences of geometric transformations, five samples, one from each focus group, were selected based on three criteria: 1- representative, 2- showing the change in students' aesthetic beliefs, and 3- demonstrating how students think logically in dealing with realistic of geometric transformations. The samples selected and discussed from each group were not fundamentally different from the others.

Table 2. Codes related to students' aesthetic experiences in dealing with the realistic activities of geometric transformations during dialogue, observing, and interviewing

Selective codes	Axial codes	Open codes
The visual beauty of mathematics and pleasant visualizations	Order and harmony	Axial reflection or symmetry, like numbers, has order and harmony
	Simplicity and complexity	Geometric transformations can look simple yet complex when put together in a pattern
	Unity, proportionality, and repeatability	Geometric transformations activities follow a repeatable process and form a uniform and proportional pattern
	Color and shape	When activities of geometric transformations are presented along with colored shapes, they attract attention and become more interesting
Aesthetic emotions and feelings	A sense of self-confidence and active engagement	Discussion and manipulation in the group create a sense of empowerment and freedom from helplessness to solve the activity
	Criticism	The process of finding an answer by hearing the opinions of others and confirming or rejecting them strengthens a spirit of participation and criticism.

	The joy of interactive learning	Sharing the final answers among the groups creates an experience of success and collective wisdom
Aesthetic usefulness in individual and collective life	Connection with everyday life	Images and patterns similar to those seen in daily life and experienced in transformations are amazing.
	Application in life and society	The floor parquetry activity reminds and evokes the pattern of geometric transformations in the scenery of the city, carpet patterns, paintings, and architectural works.
	Inspiration in life	Encountering realistic activities in geometric transformations will bring inspiring experiences to deal with realistic activities .
	Universality	Most people use them regardless of geometric transformations in dealing with some activities.
Aesthetic connection and mathematical thinking	Thinking differently (intellectual beauty)	Different and even sometimes contradictory ideas in performing realistic activities of geometric transformations are interesting and thought-provoking.
	Understanding concepts and reasoning	Depicting geometric shapes and their reproducibility in transformations provides the possibility of understanding the concepts and the path to reasoning and logical thinking.
	Dialogue and logical thinking	Presenting different illustrating to do the activity, evaluating them, and reaching a consensus in a group experience leads to logical thinkin g.
	Meta-analysis and generalization)abstract beauty(	In addition to intuitive, geometric, and computational solutions, reasoning solutions in the field of algebra beyond geometric transformations also appear.

Students' beliefs about the aesthetic experiences of geometric transformations

Emotions, beliefs, and judgments about beauty are highly personal, and a comprehensive aesthetic definition of a geometric structure is difficult (54). However, realizing beliefs can be attained by experiencing the beliefs in the reasoning environment (38). Brinkmann (2009) showed that revealing aesthetic features within mathematics is a criterion for achieving mathematical aesthetic experience (48). Parrish (2009), in his five principles, emphasized the essential role of dialogue in revealing students' beliefs and creating learning opportunities (2). To determine the students' aesthetic beliefs, we analyzed students' dialogue and conducted semi-structured interviews considering some items of Brinkman's questionnaire (Table 3). All students' names in this research are pseudonyms.

Students' logical thinking skills about the aesthetic experiences of geometric transformations

Every logical thinking process has three primary steps: assignment, diagnosis, and problem-solving (24). As stated in the methodology section, the educational course focused on four activities of geometric transformations. During the implementation of the course, we collected and analyzed data related to students' logical thinking. Five representatives, who could change their role with each member of their focus group, showed different aesthetic experiences and logical thinking in dealing with the activities of geometric transformations.

Hereafter, we present excerpts from the dialogues and interviews for the four activities.

Table 3. Indexing students' aesthetic experiences of geometric transformations and aesthetic beliefs

Representative of the focal group	How do you depict the beauty of geometric transformations?)The aesthetic nature of transformations(	Express your belief and previous experiences about the beauty of geometric transformations (Dispositional beliefs)	Has the experience of facing activities of geometric transformations changed your belief about the beauty of geometry? (Occurrent beliefs)
Armita	It emphasizes the beauty of proving theorems in depicting the beauty of geometric transformations.	I think the beauty of geometry lies in geometry itself: geometric transformations and proofs.	The experience of learning transformations has made me think that geometric transformations can simplify the path to some proofs.

Diba	Depicting a dummy with simple geometric shapes and their reflection	I am sure that geometric transformations are very beautiful and I can contribute to making them more beautiful.	The beauty of architectural works of art and their mental visualizations have strengthened my opinion about the beauty of geometrical transformations.
Tina	Depicting congruent and nested circles with a marker and colored pencil	I liked geometry and geometric transformations from the very beginning of school. The more complicated a mathematics activity, the better and more beautiful.	I am starting to think that even geometric transformations that do not observe symmetry have amazing beauty!
Mahsa	Depicting the beauty of transformations in the symmetry, order, and arrangement of the petals of a flower.	I have always liked tinkering with geometric shapes, especially paper crafts. I think geometric transformations are wonderful in making paper lanterns.	I am sure that the beauty of geometry is not only related to the essence of the shape itself but the beauty of transformations can be learned and understood with attractive tutorials.
Sara	Depicting a yard with a symmetrical area	Disorder and disorganization make my mind tired and confused. I like everything symmetrical and orderly. I always arrange the furniture in my room so that they are symmetrical.	The beauty and good feeling resulting from the order governing the geometric transformations strengthened my belief about how "geometry can be effective in life and creating intellectual order"!

Activity 1: The clay tiles

Question: In the archaeological excavations, fragments of a clay tile were found (Figure 2). According to historical research, it was determined to have a shape with at least two axes of symmetry. What do you think the form of the tile looks like?

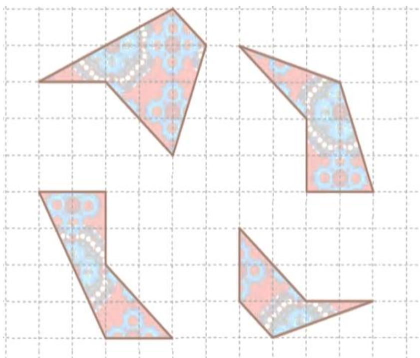


Figure 2. Pieces of the clay tiles

Armita and Diba, two representatives from two groups, looked for symmetrical shapes, such as squares, rhombi, and rectangles, in dealing with the clay tiles activity.

They said that they have seen similar ones in tiling and the tiles in Figure 2 are beautiful as these geometric shapes have an axial reflection. They could not identify all conditions of the activity to provide a proper logical reasoning.

Mahsa and Sara, two representatives from two groups, copied the pattern of the tiles with the help of colored papers and were looking for a solution by putting them together. They thought that colors in geometric transformations create extraordinary images, and these colored images can play a role similar puzzle to reach the solution and way of thinking and reasoning.

Tina and her group were looking for the solution by counting the squares of the checkerboard in the tile pieces and putting the pieces together according to the proportions and shape of the tile background. They said that numbers are one of the wonders in mathematics, which we can use to solve many problems by calculation. They carefully went through the step of identifying the goal and conditions of the activity. During the third step, they responded correctly to the activity using logical reasoning and mental reflection (Figure 3).

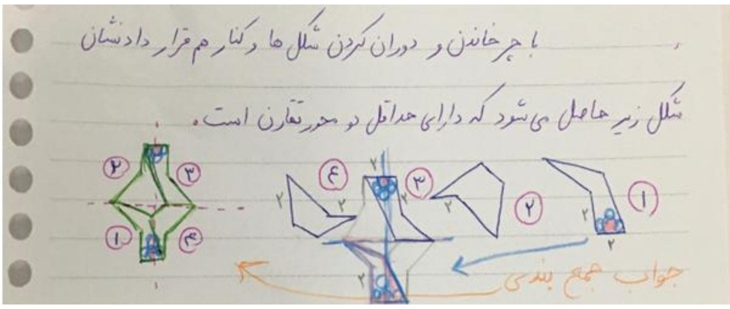


Figure 3. Tina and her group's solution to the clay tile activity

Activity 2: The game table

Question: In a game, there is a rectangle table with 8 and 5 unit dimensions. This table has four holes in the corners. In the ways depicted in Figure 4, the ball may be shot from points A, B, or C. Which point(s) are appropriate if we want the ball to enter one of the holes after striking the table's walls a maximum of six times?

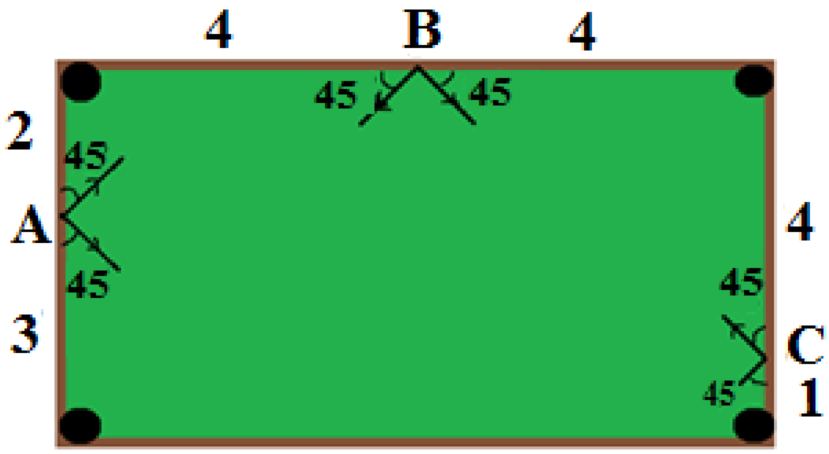


Figure 4. The game table activity and throwing the ball adopted from the pool table problem (42)

The data gathered from the implementation of the game table activity, the dialogue between student-student and student-teacher and the interview revealed aesthetic experiences such as: understanding the intellectual beauty, the usefulness and efficiency of transformations in games, the aesthetic evaluation of reasoning, and the feeling of appreciation for mathematics.

Group A:

Teacher: Where did you start to answer the question? Why?

Armita: We started from point B. If we check one side of B, the result for the other side will be the same because of its symmetry property and repeatability.

Teacher: Do you think this point is suitable for throwing the ball? What is your reason?

Armita: Yes. Considering the property of reflection, since it is the angle of the throw, it returns with the same angle, and the ball is placed in the hole before six hits according to the shape and path of the reflection (Figure 5).

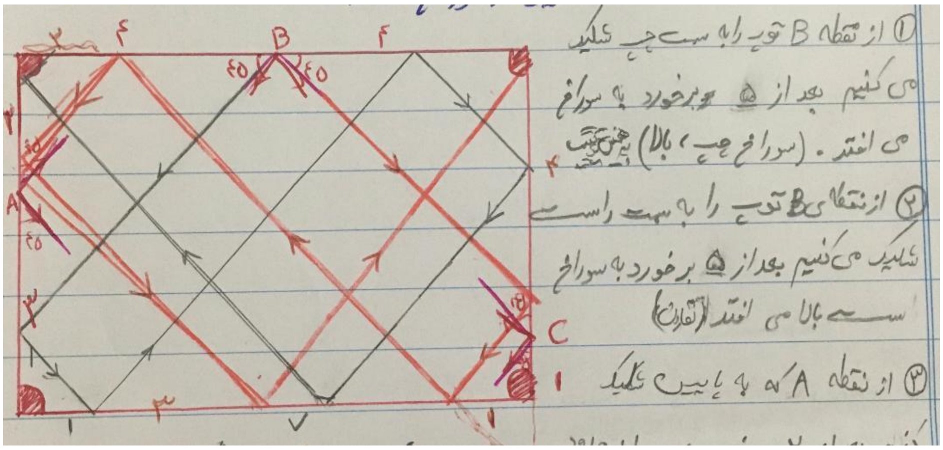


Figure 5. Armita's group's explanation about the activity of the game table and throwing the ball

Although this group achieved a wrong answer during the logical thinking process, the teacher told them they got a different, valuable answer. She encouraged this group to dialogue with others.

Teacher: Do you want to check other points as well? Why?

Armita: Yes, because we may achieve the goal by fewer hitting the wall. We must check throwing the ball from both sides of A and C.

Group B and C:

Diba and Tina said that the probability of point A being the correct answer is higher than points B and C, because of the property of reflection and the summation of 3 and 5 equals 8. Their logical thinking clarified finding the correct answer promptly (Figure 6).

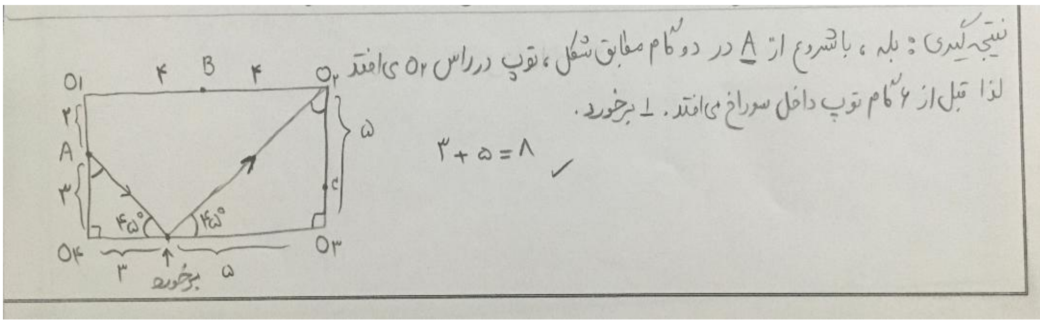


Figure 6. Diba's group's illustration about the activity of the game table and throwing the ball

Teacher: Do you think points B and C need to be checked?

Tina: Yes, we should also check them.

Diba: No, there is no need to check point's B and C. Because their conditions are not like point A, and the ball will not fall into the hole once it hits the wall.

Using their logical thinking, both groups could conclude the correct answer for point A. However, Tina's group understood the goal of the activity correctly, while Diba's group considered finding the optimum answer.

Group D:

Mahsa and her group started from point C randomly without considering the conditions and stated that point C could be the answer. When they checked points A and B, they concluded that these points could not be the correct answers. Finally, they selected point C as the right answer.

The group members could not present a reasonable dialogue without considering the goal and conditions of this activity.

Group E:

Sara and her group said that because the table's shape is rectangular and does not have central symmetry, it is better to start from points A or C. They said that point B, which is symmetrical, seems inappropriate.

The results indicate that considering several aesthetic elements in transformations (symmetry, proportion and repeatability) has a significant role in students' logical thinking process. Also, the role of the beauty of thought and logical thinking is revealed in reasoning and, thus, the speed of reaching the proper answer.

Activity 3: The floor parquetry

Question: The following design is a type of floor parquetry (Figure 7). Determine the set of rotations for this parquetry.

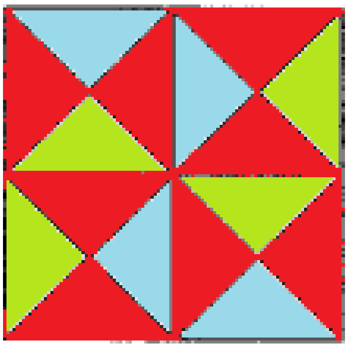


Figure 7. The floor parquetry activity

Thematic analysis from reviewing the videos of students' dialogues and interactions in the floor parquetry activity revealed various ideas, visual beauty and pleasant visualization. According to Eberle (2014), aesthetics motivates students to think about the necessary ideas for performing the activity while they do not entirely grasp it. Results showed that engaging students in the mathematical intellectual experience and the different stages of the logical thinking process creates more emancipation and ideas for students to discuss while looking for reasons, as Dewey (1900) mentioned about doubt, mathematical paradox and turning the discussion into reasoning.

We provide excerpts from dialogues and interactions that the focus groups had in dealing with the floor parquetry activity as follows:

Logical thought with doubts

Armita: Friends! What point should we consider for the center of the rotation?

Sarina: Can we compare one of the small squares as a base with the rest? (Figure 8)

Figure 8. Displaying the rotations of the floor parquetry activity

Fateme: I speculate it is better to consider the center of the original shape as the center of the rotation.

Sahar: I guess we should check both statuses.

Jina: What about if each corner of the primary shape can also be considered as the center of the rotation?

Mathematical paradox: simplicity versus complexity

Diba: It seems simple: roll the parquetry on the ground and check the different statuses.

Sima: Do you mean to rotate the shape around its center? Don't we need to rotate one of its corners?

Mahsa: It got complicated. Should we consider the direction of rotation clockwise or counter-clockwise?

Hadis: I think we should check both clockwise and counter-clockwise.

Tara: Both are the same. The angle of rotation is essential.

Visual beauty and pleasant visualization in creating various ideas

Sara: When I look at it from a distance, the rhombus in the middle of the parquetry looks like a carousel rotating.

Tarane: So, should we consider the center of the rotation in the middle of the shape?

Azin: I agree with Sara. The small triangles around the center of the shape are rotating at a 90° angle.

Sevda: If we only imagine the red colors which are placed between green and blue triangles, we can see the rotation's angle which is 90°.

Lily: Are the rotation angles for green and blue triangles the same as the red ones?

Activity 4: The wall knot

Almost all students have encountered the necessity of making knots in their daily life and have experienced the skill of knotting. The application of geometric transformations, especially the rotation, in the production of the knot is determined with a closer look at the structure of a simple knot. We used the wall knot activity from Larson et al. (2004). We provide an excerpt of the students' dialogues and interactions about the wall knot activity and the knotting steps in Table 4.

Table 4. Excerpt from students' dialogues in performing the wall knot activity

Focus groups	Students' dialogues
Group A	To make a wall knot, we need three ropes, one rope is the center of the rotation and fixed, and the other two ropes create the knot by rotating around that fixed rope.

Group B	I think we should first remember to tie the knot with the help of our shoelaces practically. We have done this many times but never looked at it like this. It is interesting, while the ropes rotate one after the other, the point which is the center of the rotation remains fixed. The center of rotation here is the woven base of the ropes! I think each of the ropes rotates about the size of a semicircle (180°) around the center, which is the woven base of the ropes. I have the same opinion.
Group C	We have done the knotting many times and it is very simple. We concluded that the knot is simply created by rotating the ropes around each other.
Group D	We considered each part of the knot shape separately: Three rotations take place by the ropes to create a knot. One rope rotates by 90° and two other ropes rotate by 180°
Group E	The wall knot evoked hair weaving in our mind We waved Negin's hair and wrote the knotting steps Each of the three separate parts of the Negin's hairs was rotated around the central fixed point 90°.

As can be seen from this table, the results confirmed the role of aesthetics, sensory experiences in problem-solving, the joy of free exploration, and the appreciation of the usefulness of mathematics and its relationship with everyday life.

An instance of the students' dialogues in Figure 9 depicts the logical thinking and reasoning process used to describe the concept of the knot wall. Some have taken assist from practical experiences, such as tying shoelaces and hair braiding, to illustrate their thoughts and reasonings.

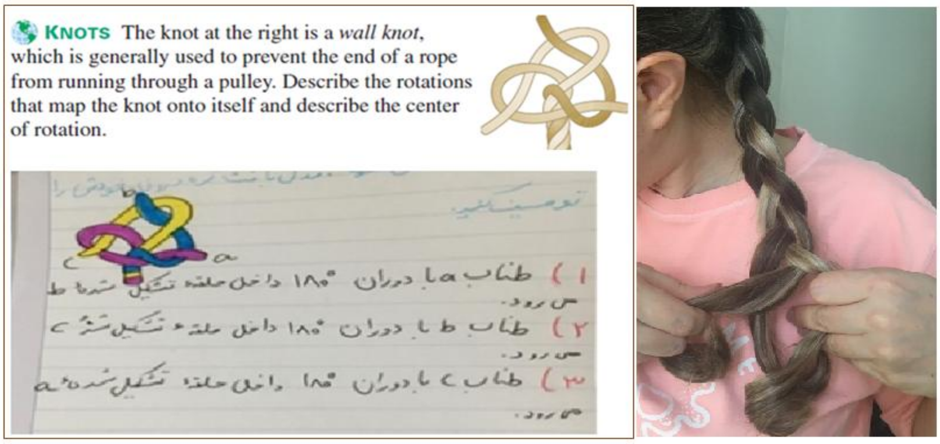


Figure 9. The wall knot activity (55) and a sample of students' solution by practical experience

Students with hair braiding experience could recognize the activity's aim, considering the limitations and conditions. They developed ideas systematically during the practical experience. After discussing their reasons, they went through the logical thinking stages until reaching a rational answer.

Discussion and Conclusion

The current study examined how aesthetic experiences derived from realistic activities of geometric transformations can shape high school students' beliefs about mathematical beauty and enhance their logical thinking processes. Through Charmaz's grounded theory approach and thematic analysis, the findings revealed that students, when immersed in aesthetically enriched mathematical tasks, developed nuanced aesthetic beliefs and demonstrated logical reasoning patterns beyond rote application. These outcomes point to the pedagogical potency of integrating aesthetic principles in mathematics education—particularly in geometry—to foster deeper engagement and conceptual understanding.

The aesthetic experiences observed in this study encompassed elements such as visual beauty, emotional pleasure, confidence, everyday relevance, and reflective dialogue. These components align with Uhrmacher's model of aesthetic learning, which emphasizes interaction, sensory perception, and imagination as core ingredients of transformative educational experiences (5). Students who initially viewed geometry as a rigid discipline began to appreciate its structure, coherence, and visual harmony through tasks like tile reflection, floor parquetry, and knot rotation. The activation of both dispositional and occurrent beliefs during these activities reflects Pitt's argument that beliefs can become vivid and actionable when contextualized within authentic experiences (37). Moreover, the process of belief change observed among students supports Potrč's notion that latent dispositions become cognitively operative when aligned with appropriate contextual stimuli (38).

One of the most compelling findings was how aesthetic triggers served as cognitive entry points into logical thinking. Students navigated tasks with increasing complexity by shifting from intuitive or trial-based strategies to deductive reasoning. This progression echoes Dewey's classic model of logical thought—assignment, diagnosis, and problem-solving—as students moved from identifying visual patterns to articulating hypotheses and verifying solutions (24). In the game table activity, for instance, students discussed symmetry, reflection, and angles as criteria for evaluating the optimal ball trajectory, illustrating how aesthetic insights can scaffold abstract reasoning. This finding affirms the role of beauty in prompting analytical thinking, as theorized by Sinclair, who argued that aesthetic appreciation can be generative, evaluative, and motivational in mathematical inquiry (3).

The aesthetic-emotional dimension of learning also played a vital role. Students described feelings of joy, empowerment, and curiosity when engaging in geometric transformation activities, particularly when they involved visual symmetry, color, or real-world analogies. These responses are consistent with Koichu et al.'s emphasis on surprise and aesthetic insight as facilitators of deeper mathematical engagement (13). Similarly, the aesthetic flow described by Richman et al. during captivating math lessons appeared in the form of heightened student attentiveness and prolonged engagement in exploratory discussions (6). These outcomes also corroborate the work of Gadanidis et al., who advocate for designing low-floor, high-ceiling tasks that offer cognitive accessibility while inviting aesthetic exploration (7).

Importantly, the activities encouraged students to articulate their beliefs about the nature of beauty in mathematics. As seen in the interviews and focus group discussions, students referenced order, symmetry, proportionality, and repetition as qualities contributing to the beauty of geometric transformations. These findings reflect broader philosophical and psychological theories of aesthetics, including Reber et al.'s fluency theory and Montano's assertion that mathematical beauty emerges from the interrelatedness of ideas and structural harmony (22, 26). Students who once saw mathematics as merely procedural began to express a newfound appreciation for its visual and logical coherence—a shift that has profound implications for identity development and motivation in mathematics learning (4).

Moreover, this transformation occurred not only at the individual level but also through collaborative meaning-making. The focus group interactions revealed that the aesthetic experiences were often co-constructed through dialogue, critique, and shared interpretation. These social processes align with the findings of Wanzer et al., who emphasized the communal nature of aesthetic experience, and Hobbs, who underscored the interconnection between teacher identity, passion, and aesthetic teaching practices (16, 39). In this study, students' peer discussions about geometric patterns, angles, and reflections served as both cognitive and affective scaffolds that enhanced their reasoning processes and deepened their engagement.

The findings also suggest that aesthetic experiences play a role in democratizing mathematics education. Students with different learning preferences or prior attitudes toward math found entry points through the visual, tactile, and cultural dimensions of the tasks. For instance, the wall knot activity invoked personal experiences like hair braiding and shoelace tying, which allowed students to relate abstract mathematical ideas to lived realities. This observation aligns with the claims of Hafezi et al., who emphasized the value of cultural and artistic education in making academic content relatable and meaningful (44), and with Zhou's research on how artistic activities can bridge social and emotional gaps in educational settings (31).

At the curricular level, these results advocate for integrating aesthetics as a core design principle in mathematics instruction, particularly in geometry. The research shows that aesthetic experiences are not merely supplementary but foundational in supporting the cognitive, emotional, and social dimensions of mathematical learning. The theoretical models of Parrish and Uhrmacher, alongside the empirical evidence from this study, provide a compelling framework for educators seeking to enrich their practice with aesthetic and culturally responsive pedagogy (2, 5). Furthermore, the research contributes to the growing body of literature that positions aesthetics as an epistemological tool in mathematics education—one that helps learners construct, validate, and communicate mathematical understanding (9, 12).

Despite the promising findings, this study is not without limitations. First, the sample was limited to female students from a single high school in Tehran, which may restrict the generalizability of the results. The cultural, gender-based, and contextual nuances of aesthetic perception and engagement in mathematics may vary across different student populations. Second, the duration of the intervention, although sufficient for observing immediate transformations in beliefs and reasoning, may not capture the long-term retention or transfer of these experiences. Third, the reliance on qualitative self-report methods, such as interviews and group discussions, may introduce subjective biases. While triangulated with observations and document analysis, future studies would benefit from integrating longitudinal and quantitative measures of belief change and cognitive development.

Building upon this foundational inquiry, several avenues for future research are proposed. First, similar studies should be conducted with more diverse samples, including male students, students from rural regions, and those with special educational needs, to explore whether the aesthetic-laden approach to geometric transformations yields consistent results across populations. Second, longitudinal research could track the stability of students' aesthetic beliefs and logical reasoning skills over time, especially in subsequent mathematical domains such as algebra or calculus. Third, interdisciplinary studies combining insights from neuroscience, psychology, and education could investigate the neural and cognitive mechanisms underlying aesthetic appreciation in mathematical thinking. Additionally, action research with teachers implementing these principles in live classrooms would yield practical insights and foster co-constructed knowledge between researchers and practitioners.

To translate these findings into actionable strategies, several pedagogical recommendations can be made. Teachers should deliberately design mathematical tasks that contain aesthetic elements—such as symmetry, harmony, color, and cultural motifs—to foster engagement and cognitive curiosity. These tasks should encourage open-ended exploration, allow multiple entry points, and integrate student experiences to make learning more personal and relevant. Educators should facilitate reflective dialogues that prompt students to express their feelings, perceptions, and judgments about beauty in mathematics. Teacher preparation programs should include training in aesthetic learning design and culturally responsive pedagogy to help educators recognize and harness the affective dimensions of mathematical experience. Finally, school curricula and policy frameworks should validate and support the inclusion of aesthetic goals alongside cognitive and procedural outcomes in mathematics education.

Acknowledgments

We would like to express our appreciation and gratitude to all those who helped us carrying out this study.

Authors' Contributions

All authors equally contributed to this study.

Declaration of Interest

The authors of this article declared no conflict of interest.

Ethical Considerations

All ethical principles were adheried in conducting and writing this article.

Transparency of Data

In accordance with the principles of transparency and open research, we declare that all data and materials used in this study are available upon request.

Funding

This research was carried out independently with personal funding and without the financial support of any governmental or private institution or organization.

References

1. Diana D, Turmudi, Yohannes, editors. Analysis of teachers' difficulties in implementing STEM approach in learning: a study literature2021.
2. Parrish PE. Aesthetic principles for instructional design. Educational Technology Research and Development. 2009;57(4):511-28. doi: 10.1007/s11423-007-9060-7.
3. Sinclair N. Aesthetics as a liberating force in mathematics education? ZDM-The International Journal on Mathematics Education. 2009;41(1):45-60. doi: 10.1007/s11858-008-0132-x.
4. Chen RJ. Prospective elementary teachers' aesthetic experience and relationships to mathematics. Journal of Mathematics Teacher Education. 2017;20(3):207-30. doi: 10.1007/s10857-015-9329-4.
5. Uhrmacher PB. Toward a Theory of Aesthetic Learning Experiences. Curriculum Inquiry. 2015;39(5):613-36. doi: 10.1111/j.1467-873X.2009.00462.x.
6. Richman AS, Dietiker L, Riling M. The plot thickens: The aesthetic dimensions of a captivating mathematics lesson. The Journal of Mathematical Behavior. 2019;54:100671. doi: 10.1016/j.jmathb.2018.08.005.
7. Gadanidis G, Borba M, Hughes J, Lacerda H. Designing aesthetic experiences for young mathematicians: A model for mathematics education reform. International Journal of Research in Mathematics Education. 2016;6(2):225-44.
8. Huser J. STEAM and the Role of the Arts in STEM. 2020.
9. Rieger A. The Beautiful Art of Mathematics. Philosophia Mathematica. 2017;26(2):234-48. doi: 10.1093/philmat/nkx006.
10. Saxe AB, Wilson JL. Curriculum, aesthetics, and social justice: From the common to the exceptional. Current Issues in Education. 2021;22(2):1-20.

11. Fife JH, James K, Bauer M. A Learning Progression for Geometric Transformations. ETS Research Report Series. 2019;2019(1):1-16. doi: 10.1002/ets2.12236.

12. Papageorgiou E, Xenofontos C. Discovering geometrical transformations in the ancient mosaics of Cyprus: An instructional approach to Grade 6. Australian Mathematics Teacher. 2018;74(2):34-40.

13. Koichu B, Katz E, Berman A. Stimulating student aesthetic response to mathematical problems by means of manipulating the extent of surprise. The Journal of Mathematical Behavior. 2017;46:42-57. doi: 10.1016/j.jmathb.2017.02.005.

14. Tjoe H. Aesthetics in School Mathematics: A Potential Model and A Possible Lesson. The Mathematics Enthusiast. 2016;13(3):279-302. doi: 10.54870/1551-3440.1378.

15. Mahdavi A, Darabi F, Ahmadinia H. Investigating the relationship between spiritual intelligence and aesthetic intelligence with suicidal thoughts among students of Asadabad Medical Sciences Faculty: A descriptive study. Scientific Journal of Rafsanjan University of Medical Sciences. 2023;22(6):581-96. doi: 10.61186/jrums.22.6.581.

16. Wanzer DL, Finley KP, Zarian S, Cortez N. Experiencing flow while viewing art: Development of the Aesthetic Experience Questionnaire. Psychology of Aesthetics, Creativity, and the Arts. 2020;14(1):113-24. doi: 10.1037/aca0000203.

17. Huang Shih Y. Teaching Principle for Aesthetic Education: Cultivating Taiwanese Children's Aesthetic Literacy. International Journal of Education and Practice. 2020;8(3):568-76. doi: 10.18488/journal.61.2020.83.568.576.

18. Yang CP. Hepburn's natural aesthetic and its implications for aesthetic education. International Education Studies. 2013;6(7):225-31. doi: 10.5539/ies.v6n7p225.

19. Moroye CM, Uhrmacher PB. Teaching in the moment: Educational experience in the age of tomorrow. On the Horizon. 2018;26(2):170-80. doi: 10.1108/OTH-02-2018-0009.

20. Stemhagen K, Henney C. Democracy and Mathematics Education (Rethinking School Math for Our Troubled Times): Routledge; 2021.

21. Parlak Sert H, Başkale H. Students' increased time spent on social media, and their level of coronavirus anxiety during the pandemic, predict increased social media addiction. Health Information & Libraries Journal. 2023;40(3):262-74.

22. Montano U. Explaining Beauty in Mathematics: An Aesthetic Theory of Mathematics: Springer; 2014.

23. Portales C. Objective Beauty and Subjective Dissent in Leibniz's Aesthetics. Estetika: The Central European Journal of Aesthetics. 2018;55(1):67-88. doi: 10.33134/eeja.171.

24. Von Plato J. Elements of Logical Reasoning: Cambridge University Press; 2013.

25. Piirto J, Baska B. Serving Artistically Gifted Children and Youth Handbook for Counselors Serving Students With Gifts and Talents: Routledge; 2021. 231-46 p.

26. Starikova I. Aesthetic Preferences in Mathematics: A Case Study. Philosophia Mathematica. 2018;26(2):161-83. doi: 10.1093/philmat/nkx014.

27. Guven B. Using dynamic geometry software to improve eight grade students' understanding of transformation geometry. Australasian Journal of Educational Technology. 2012;28(2):364-82. doi: 10.14742/ajet.878.

28. Ahmad M, Siregar YP, Siregar NA. The Effectiveness of Realistic Mathematics Learning Model Based on Mandailing Culture in Teaching of Students' Mathematical Problem Solving Ability. Advances in Social Science, Education and Humanities Research. 2018;285:1-. doi: 10.2991/icm2e-18.2018.31.

29. Haji Babaie HR. National Curriculum Document of the Islamic Republic of Iran. 2013.

30. Ankyiah F. When the Artist Studio Is Alive, Art Is Alive: Empowering Art Educators and Students Through Live Studio Practice. Asian Research Journal of Arts & Social Sciences. 2023;21(1):1-11. doi: 10.9734/arjass/2023/v21i1457.

31. Zhou Y. Artistic Sports Activities Effectiveness for Enhancing Students' Academic Performance Among Left-Behind Children: Mediating Effects of Loneliness. Frontiers in Psychology. 2024;15. doi: 10.3389/fpsyg.2024.1366501.

32. Nurjamaludin M, Gunawan D, Adireja RK, Alani N, editors. Realistic Mathematics Education (RME) approach to increase student's problem solving skill in elementary school2020.

33. Hirza B, Kusumah YS, Darhim, Zulkardi. Improving Intuition Skills Realistic Mathematics Education. Journal on Mathematics Education. 2014;5(1):27-34. doi: 10.22342/jme.5.1.1446.27-34.

34. Sevinc S, Lesh R. Training mathematics teachers for realistic math problems: A case of modeling-based teacher education courses. ZDM - Mathematics Education. 2018;50(1):301-14. doi: 10.1007/s11858-017-0898-9.

35. Zazkis R, Sinclair N, Liljedahl P. Lesson play in mathematics education: A tool for research and professional development: Springer; 2013.

36. Eberle RS. The role of children's mathematical aesthetics: The case of tessellations. The Journal of Mathematical Behavior. 2014;35:129-43. doi: 10.1016/j.jmathb.2014.07.004.

37. Pitt D. Conscious Belief. Rivista Internazionale di Filosofia e Psicologia. 2016;7(1):121-6.

38. Potrč M. Dispositional beliefs. Interdisciplinary Description of Complex Systems. 2015;13(4):548-57. doi: 10.7906/indecs.13.4.7.

39. Hobbs L. Examining the aesthetic dimensions of teaching: Relationships between teacher knowledge, identity and passion. Teaching and Teacher Education. 2012;28(5):718-27. doi: 10.1016/j.tate.2012.01.010.

40. Larcombe PJ. A Few Thoughts on the Aesthetics of Mathematics in Research and Teaching. Palestine Journal of Mathematics. 2018;7(1):1-8.

41. Dewey J. Some Stages of Logical Thought. The Philosophical Review. 1900;9(5):465-89. doi: 10.2307/2176692.

42. Charles RL, Hall B, Kennedy D, Bass LE, Johnson R, Murphy SJ, et al. Geometry Common Core Textbook: Pearson Education; 2015.

43. Denac O. The Significance and Role of Aesthetic Education in Schooling. Creative Education. 2014;5(19):1714-9. doi: 10.4236/ce.2014.519190.

44. Hafezi A, Abedipour MS, Khosravi H, Fattahpour A, Sharifi S, editors. Cultural and Artistic Education of Students. Fifth International Conference on Educational Sciences, Psychology, Counseling, and Education; 2025.

45. Mansouri SA. From Aesthetic to Aesthetics. Journal of Art and Civilization of the Orient. 2018;5(18):3-6.

46. Grütter JK. Asthetik der Architektur Grundlagen der Architektur-Wahrnehmung: W. Kohlhammer; 1987.

47. Karp AP. Which problems do teachers consider beautiful? A comparative study. For the Learning of Mathematics. 2008;28(1):36-43.

48. Brinkmann A. Mathematical Beauty and its Characteristics- A Study on the Students' Points of View. The Mathematics Enthusiast. 2009;6(3):365-80. doi: 10.54870/1551-3440.1158.

49. Braun V, Clarke V. Using thematic analysis in psychology. Qualitative Research in Psychology. 2006;3(2):77-101. doi: 10.1191/1478088706qp063oa.

50. Braun V, Clarke V. Thematic analysis. In: Cooper H, editor. APA handbook of research methods in psychology: Research designs: Quantitative, qualitative, neuropsychological, and biological2012. p. 57-71.

51. Silverman D. Doing Qualitative Research: A Practical Handbook: Sage; 2013.

52. Miles MB, Huberman AM. Qualitative Data Analysis: An Expanded Sourcebook: Sage; 1994.

53. Lincoln YS, Guba EG. Naturalistic Inquiry: Sage; 1985.

54. Hofstadter DR. Gödel, Escher, Bach: An Eternal Golden Braid: Basic Books Inc.; 1979.

55. Larson R, Boswell L, Stiff L. Geometry - Teacher's Edition: McDougal Littell, Houghton Mifflin Company; 2004.